Errors in Measurement in 5 easy Steps

They say to err is Human … Yes, errors occur during measurements. For the best results, we should know the sources of errors and how to deal with errors.

We cover,
(i) What is an error?
(ii) What is meant by accuracy and precision;
(iii) Types of Errors
(iv) Measurement of Errors
(v) Combination of Errors

Error

◊ The uncertainty in measurement of an instrument is known as **error**.

◊ Error is the difference between actual value of a physical quantity and the observed value in an experiment.

Accuracy and Precision

◊ Two terms: **Accuracy** and **Precision**.

◊ **Accuracy** of a measurement is a measure of how close the measured value is to the true value of the quantity.

◊ The limit or resolution to which a physical quantity is measured by a measuring instrument is known as **precision**.

◊ We have two measuring instruments.

(i) **Instrument A** has a **resolution of 0.1 cm** and **Instrument B** has a **resolution of 0.01 cm**.
(ii) Suppose the true value of length of an object is 4.658 cm.
(iii) Using Instrument A, measured value is found to be 4.5 cm, while Instrument B, the length is determined to be 4.38 cm.
(iv) The **first measurement has more accuracy** (because it is closer to the true value) but less precision (its resolution is only 0.1 cm), while the **second measurement is less accurate but more precise**.

Types of Errors

◊ Errors in measurement are broadly of two types:
(a) systematic errors; and
(b) random errors.

◊ **Systematic errors** are those errors that tend to be in one direction, either positive or negative and are due to (i)Instrumental Errors; (ii) Experimental technique; and (iii) Personal Errors.

(i) **Instrumental errors** arise due to imperfect calibration of the measuring instrument. They are also called zero error in the instrument. For example, if a thermometer is wrongly calibrated, then it may read boiling point of water as 103 °C ***instead of*** 100 °C, which is the true value. This is an Instrument Error.

(ii) **Imperfection in experimental technique or procedure:** Suppose you want to measure current flowing in circuit using an Ammeter. But instead of connecting ammeter in series with the circuit you connect it in parallel to the circuit. Then there will be error in observation due to faulty setup due to imperfection in experimental technique.

(iii) **Personal errors** are due to an individual's bias or carelessness. For example, if you are careless then in the experiment for reflection of light, the angle of incidence will never be equal to the angle of reflection.

◊ Systematic errors can be **minimised** by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.

◊ **Random errors** are those errors, which occur irregularly due to random and unpredictable fluctuations in experimental conditions or personal (unbiased) errors by the observer. A classic example is when observations are repeated by the same person, we get different reading, whereas, it should have been the same reading.

◊ **Least count error** is a special kind of random errors. The smallest value that can be measured by the measuring instrument is called its **least count** and all measured value will be valid to that least count. All the readings or measured values are good only up to this value. Using a meter scale to metre scale to measure the diameter of a wire, will lead to least count error. Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error.

Measurement of Errors

◊ We explore - Absolute Error, Relative Error and Percentage Error

◊ **Absolute error** is the difference between the true value of the quantity and the individual measurement value.

(i) To find the true value, we calculate arithmetic mean of all the observation. The **arithmetic mean** is assigned as **true value** of the physical quantity.

(ii) After finding the true value, we find the difference between true value and the observation. We add all the differences so obtained. This **value is absolute error**.

◊ Relative error is the **ratio of** the **mean absolute error** to the **mean value** of the quantity measured.

(i) We know how to find the value of absolute error. When this absolute error is divided by the number of observation, we have mean absolute error.

(ii) Mean value is the true value of the physical quantity.

(iii) By dividing mean absolute error by mean value we will get relative error.

◊ **Percentage error** is Relative Error multiplied with 100.

Example: The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.29m, 2.50m and 2.48m. Find the **true length**, the **absolute error** in each observation, the **relative error** and **percentage error**.

Ans: True length $= \frac{2.48+2.46+2.29+2.50+2.48}{5} = \frac{12.41}{5} = 2.28$

The absolute errors in the different measurements are

$L_1 = 2.48 - 2.48 = 0.00$ m
$L_2 = 2.48 - 2.46 = 0.02$ m
$L_3 = 2.48 - 2.49 = -0.01$ m
$L_4 = 2.48 - 2.50 = -0.02$ m
$L_5 = 2.48 - 2.48 = 0.00$ m

The absolute error $= \frac{\sum|\Delta L|}{5} = \frac{0.00+0.02+0.01+0.02+0.00}{5} = 0.01$ m

Relative Error $= \frac{0.01}{2.48} = 0.0040$

Percentage Error $= \frac{0.01}{2.48} \times 100 = 0.40\%$

Combination of Errors

◊ Density is the ratio of mass to volume. If there are errors in the measurement of mass and of the sizes/dimensions of volume, then there will error will be in the density. To make such estimates, we should know how to combine errors in various mathematical operations like:

(i) Error of a sum or a difference;
(ii) Error of a product or a quotient; and
(iii) Error in case of a measured quantity raised to a power

◊ When two quantities are added, or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

♥ When we add, or subtract two quantities then their total error is equal to **absolute errors added up**

◊ When two quantities are multiplied, or divided, the relative error in the result is the sum of the relative errors in the multipliers.

♥ When we multiply, or divide two quantities their total errors is equal to **relative errors added up**

◊ The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

♥ When a quantity is raised to the power k, then total error is equal to **k multiplied to relative errors add up**

Example: Two resistances $R_1 = 100 \pm 3\ \Omega$ and $R_2 = 200 \pm 4\ \Omega$ are connected in series. What is their equivalent resistance?

Ans: Equivalent resistance

$R = R_1 + R_2 = (100 \pm 3) + (200 \pm 4)$
$= (100+200) \pm (3+4)$
$= (300 \pm 7)\ \Omega$

Example: The resistance $R = \frac{V}{I}$, where $V = 100 \pm 5$ V and $I = 10 \pm 0.2$ A. Find the percentage error in R

Ans: The percentage error in V is 5% and in I is 2%.
The total percentage error in R is given by

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 = 5\% + 2\% = 7\%$$

Example: The error in the measurement of radius of a sphere is 2%. What would be the error in the volume of the sphere?

Ans: Given $\frac{\Delta r}{r} \times 100 = 2\%$

Volume of the sphere, $V = 4/3\pi\ r^3$

Percent error in volume $= \frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100 = 3 \times 2 = 6\%$