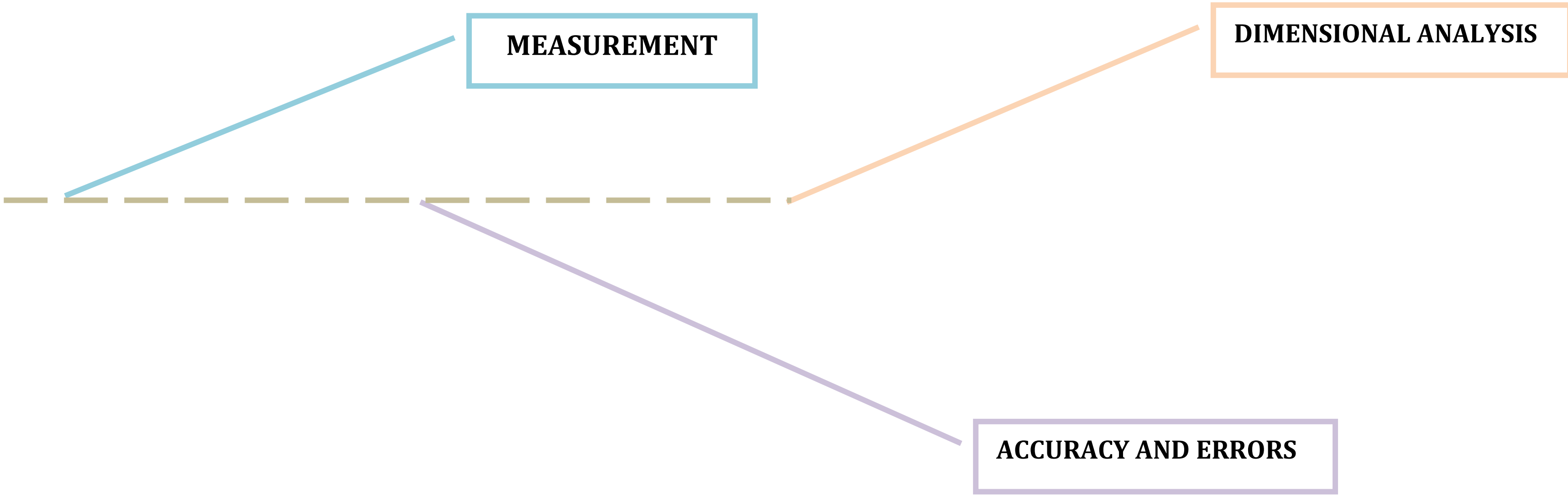
Chap2: UNITS AND MEASUREMENT

We learn three broad concepts – Measurements, accuracy & errors and dimensional analysis



MEASUREMENT

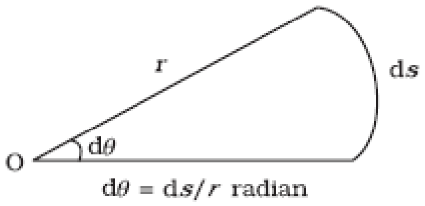
Measurement of any physical quantity on the basis of an internationally accepted reference standard called **unit**. The units for the fundamental or base quantities are called **fundamental units**. The units of all other physical quantities can be expressed as combinations of the fundamental units called **derived units**. A complete set of these units is known as the **system of units**. In SI, there are seven base units. These 7 fundamental units are Length, Mass, Time, Electric Current, Thermodynamic Temperature, Amount of Substance and Luminous intensity.

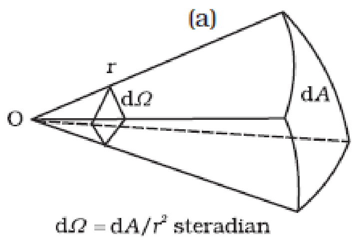
Fundamental Quantity	Name	Symbol
Length	Metre	m
Mass	Kilogram	Kg
Time	Second	S
Electric Current	ampere	A
Thermodynamic Temperature	Kelvin	K
Amount of Substance	Mole	Mol
Luminous intensity	candela	Cd

Besides the seven base units, there are two more units that are defined for

(a) plane angle $d\theta$ as the ratio of length of arc ds to the radius r and

(b) solid angle $d\Omega$ as the ratio of the intercepted area dA of the spherical surface, described about the apex O as the centre, to the square of its radius r.





Measurement of Large Distances – Parallax method

An important method for measuring large distances is the **parallax method**.It can be used to calculate the distance of a nearby star. It suppose AB is the diameter of Earth(equal to ‘b’) and the observation is made in such a way that the angle subtended by the star is θ (measured in radians) then the distance of the star ‘D’ can be found using the formula

$$D = \frac{b}{\theta}$$

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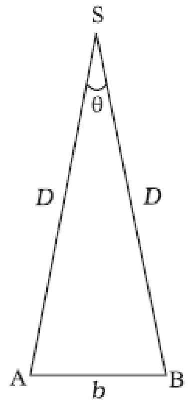

d Fig. 2.2 Parallax method.

ACCURACY AND ERRORS IN MEASUREMENT

(A) The result of every measurement by any measuring instrument contains some uncertainty called **error**. We need to understand two terms: **accuracy** and **precision**.

Accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.

Precision tells us to what resolution or limit the quantity is measured.

The errors in measurement can be broadly classified as
(a) systematic errors and
(b) random errors.

***Systematic errors* :** The **systematic errors** are those errors can either positive or negative. Some of the sources of systematic errors are :

(a) **Instrumental errors** that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(b) **Imperfection in experimental technique or procedure** To determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature.

(c) **Personal errors** that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.

Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.

***Random errors* :** The **random errors** are those errors, which occur irregularly and hence are random with respect to sign and size.

Least count error The smallest value that can be measured by the measuring instrument is called its **least count**. The **least count error** is the error associated with the resolution of the instrument.

(B) Absolute Error, Relative Error and Percentage Error with example

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the **absolute error** of the measurement.

The **relative error** is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

$$\text{Relative error} = \Delta a_{mean}/a_{mean}$$

When the relative error is expressed in per cent, it is called the **percentage error** (δa). Thus, Percentage error

$$\delta a = (\Delta a_{mean}/a_{mean}) \times 100\%$$

Example: The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.29m, 2.50m and 2.48m. Find the average length, the absolute error in each = and the relative error and percentage error.

Ans: Average length = $\frac{2.48+2.46+2.49+2.50+2.48}{5} = \frac{12.41}{5} = 2.28$

The absolute errors in the different measurements are

$\Delta L_1 = 2.48 - 2.48 = 0.00$ m
$\Delta L_2 = 2.48 - 2.46 = 0.02$ m
$\Delta L_3 = 2.48 - 2.49 = -0.01$ m
$\Delta L_4 = 2.48 - 2.50 = -0.02$ m
$\Delta L_5 = 2.48 - 2.48 = 0.00$ m

The absolute error = $\frac{\sum|\Delta L|}{5} = \frac{0.00+0.02+0.01+0.02+0.00}{5}\ \frac{0.05}{5} = 0.01$ m

Relative Error = $\frac{0.01}{2.48}$

Percentage Error = $\frac{0.01}{2.48} \times 100 = 0.40\%$

(C) Combination of Errors with example

(1) Error of a sum or a difference

When two quantities are added or subtracted, the **absolute error** in the final result is the sum of the absolute errors in the individual quantities.

$Z = A + B.$

$\pm \Delta Z = \pm \Delta A \pm \Delta B$

(2) Error of a product or a quotient

When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

$Z = AB$

$\Delta Z / Z = (\Delta A/A) + (\Delta B/B).$

(3) Error in case of a measured quantity raised to a power

Suppose $Z = A^p B^q / C^r$

Then,

$\Delta Z/Z = p\,(\Delta A/A) + q\,(\Delta B/B) + r\,(\Delta C/C).$

The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

Example: Two resistances $R_1 = 100 \pm 3\ \Omega$ and $R_2 = 200 \pm 4\ \Omega$ are connected in series. What is their equivalent resistance ?

Ans: Equivalent resistance

$$R = R_1 + R_2 = (100 \pm 3) + (200 \pm 4)$$
$$= (100+200) \pm (3+4) = (300 \pm 7)\ \Omega$$

Example: The resistance $R = \frac{V}{I}$, where $V = 100 \pm 5$ V and $I = 10 \pm 0.2$ A. Find the percentage error in R.

Ans: The percentage error in V is 5% and in I is 2%.

The total percentage error in R is given by

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 = 5\% + 2\% = 7\%$$

Example: The error in the measurement of radius of a sphere is 2%. What would be the error in the volume of the sphere?

Ans: Given $\frac{\Delta r}{r} \times 100 = 2\%$

Volume of the sphere, $V = 4/3\, \pi r^3$

Percent error in volume $= \frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100 = 3 \times 2 = 6\%$

(d) Significant Figures

Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as **significant digits** or **significant figures**.

A choice of change of different units does not change the number of significant digits or figures in a measurement.

- All the non-zero digits are significant.
- All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.
- If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant. [In 0.00 2308, the underlined zeroes are not significant].
- The terminal or trailing zero(s) in a number without a decimal point are not significant.
[Thus 123 m = 12300 cm = 123000 mm has *three* significant figures, the trailing zero(s) being not significant.]

- The trailing zero(s) in a number with a decimal point are significant.
[The numbers 3.500 or 0.06900 have four significant figures each.]

To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement in scientific notation (in the power of 10).

Rules for Arithmetic Operations with Significant Figures

(1) In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

(2) In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

Rounding off the Uncertain Digits

The *rule* by convention is that the **preceding digit is raised by 1 if the insignificant digit to be dropped (the underlined digit in this case) is more than 5, and is left unchanged if the latter is less than 5**. But what if the number is 2.745 in which the insignificant digit is 5. Here, the convention is that **if the preceding digit is even, the insignificant digit is simply dropped and, if it is odd, the preceding digit is raised by** 1.

Rules for Determining the Uncertainty in the Results of Arithmetic Calculations

If a set of experimental data is specified to n significant figures, a result obtained by combining the data will also be valid to n significant figures.
However, if data are subtracted, the number of significant figures can be reduced.

The relative error of a value of number specified to significant figures depends not only on n but also on the number itself.

Finally, remember that **intermediate results in a multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement.**

DIMENSIONS OF PHYSICAL QUANTITIES

Principle of homogeneity

The principle of homogeneity is that the dimensions of each the terms of a dimensiional equation on both sides are the same .

Any equation or formula involving dimensions (like mass, length, time , temperature electricity) have the terms with same dimensions. This helps us, therefore, to convert the units in one system to another system.

This also helps us to check a formula or the involvement of the dimensions in a formula.

The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. Note that using the square brackets [ ] round a quantity means that we are dealing with '**the dimensions of**' the quantity.

DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the *dimensional formula* of the given physical quantity.

An equation obtained by equating a physical quantity with its dimensional formula is called the **dimensional equation** of the physical quantity. Thus, the dimensional equations are the equations, which represent the dimensions of a physical quantity in terms of the base quantities.

As a practice you should know the formulas for Force, momentum, density, work, torque, strain, stress, pressure, potential energy, kinetic energy among a few others. Don't worry with a little practice you can do it all.

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